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MODELING LARGE-SCALE MIXING PROCESSES IN AN EXPANDING SUPERSONIC JET

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The existence of large-scale instability waves realizing large-scale mixing processes in supersonic turbulent jets is an important factor affecting both the flow structure and the noisemaking process therein. It is detected that such fluctuations in subsonic jets can result in the formation of coherent structures of the type of toruses, simple and double spirals, which under their further evolution will result in the generation of broadband noise and noise associated with the nonlinear development of instability waves [1].

Because of the technical complexity of their formulation, there are extremely few such experiments for high-velocity jets; consequently, many aspects of flow and instability wave interaction still have not been elucidated finally [2, 3]. In this situation it is impossible to underestimate the efficiency of mathematical modeling methods, which can contribute to the comprehension of definite stages in such an interaction. There have not been such researches for supersonic jets.

Speaking of the kind of large-scale waves that are evolutionary in a supersonic flow, it is necessary to note that the most important are the perturbations called the jet column mode which damp out both the mixing layer and the potential kernel during their development. As compared with the shear-layer mode originating at the root of the jet, they carry more energy, have a broad frequency spectrum, and are more characteristic for jets. The frequency and structural forms of such waves have been studied well enough [4-6].

Investigated in this paper are interaction processes of finite intensity perturbations of the jet-column-mode type with a design supersonic turbulent axisymmetric cold jet at its initial section. It is assumed that the fine-scale turbulence is in the energetic equilibrium state with the mean flow and exerts no influence on its development. There is examined what changes can occur in the stream under the action of unit waves of different spectral form (axisymmetric $n = 0$, and azimuthal or spinal $n = 1$ and 2) and more complex fluctuations of flapping type (the superposition of synchronized right- and left-twisted spirals $n = \pm 1$ and ± 2).

The mean velocity vector $u = |U_0, 0, W_0|$ of such a flow has both a radial U_0 and a longitudinal W_0 component. Here and henceforth, dimensionless quantities are used, the nondimensionalization is performed by dividing by $\bar{r}$ (the initial radius), and $\bar{W}$, $\bar{\rho}$ (the longitudinal velocity and density in the flow core). In the jet core ($r < r_1 = 1 - \delta/2$)$u = |0, 0, 1|$, in the external field ($r > r_2 = 1 + \delta/2$)$u = |0, 0, 0|$, and in the mixing layer of thickness δ($r_1 \le r \le r_2$) the longitudinal component is approximated by the Schlichting relationship [7]

$$W_0 = 1 - (1 - \eta^{1,5})^2,\ \eta = (1 - r + \delta/2)/\delta,\ 0 \leqslant \eta \leqslant 1.$$

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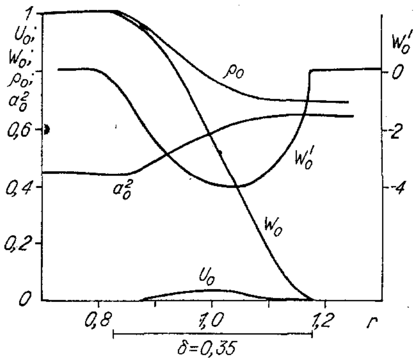

Fig. 1

The distribution W_0 is a typical inviscid unstable profile with inflection at r = 1 + 0.10315δ.

For an isobaric jet of ideal compressible gas the known gasdynamic formulas can be used and the flow parameters can be expressed in terms of the velocity W_0 and the Mach number M_0 on the axis

$$\rho_0 = [0.2M_0^2(1 - W_0^2) + 1]^{-1}, \quad a_0^2 = [\rho_0 M_0^2]^{-1},$$

and the radial component U_0 in the mixing layer can be found from the continuity equation

$$\frac{\partial}{\partial r} r\rho_0 U_0 = -r\frac{\partial}{\partial z}\rho_0 W_0 \text{ for } U_0(r=r_1)=0.$$

The law connecting δ and z [$\delta = bz$, $b = 0.135(1 + \rho_0(r_2))$] is taken from the relationship presented in [7]. The fundamental flow characteristics are given in Fig. 1 for $M_0 = 1.5$ and δ = 0.35. The spectral characteristics and structural forms of the instability waves

$$\{u_r', u_\varphi', u_z', \rho', p', s'\}(r, \varphi, z, t) = \varkappa\{iv, w, u, g, \Pi, s\}(r)e^{i\sigma_1},$$
$$\sigma_1 = \alpha z - \omega t + n\varphi,$$

where κ is the amplitude parameter, are obtained from the linearized equations describing the behavior of small perturbations in a compressible nonheat-conducting inviscid fluid [8]. As is shown in [3], this approximation describes the fluctuation shape and characteristics well up to an 8% wave intensity. There remains to add that the influence of the radial velocity U_0 and the dependence κ(z) on the wave is also not taken into account. The results in [5], where it is shown by the methods of different scales that the influence of these factors is negligible for low supersonic velocities ($M_0 \sim 1.5$), are the basis for such a simplification.

A simple finite intensity wave induces Reynolds stress in the flow, which is homogeneous in the azimuthal angle φ, where $u_\varphi' = 0$ for axisymmetric fluctuations. Let us write some of them down (averaged over the phase space σ_1):

$$\langle u_r'^2\rangle = \varkappa^2\langle v^2\rangle \exp(-2\alpha_i z)/2, \quad \langle u_\varphi'^2\rangle = \varkappa^2\langle w^2\rangle \exp(-2\alpha_i z)/2,$$
$$\langle u_r' u_z'\rangle = \varkappa^2\langle vu\rangle \exp(-2\alpha_i z)/2, \quad \langle \rho' u_z'\rangle = \varkappa^2\langle gu\rangle \exp(-2\alpha_i z)/2.$$

The real part of the corresponding complex amplitudes

$$\langle vu\rangle = v_r u_i - v_i u_r, \quad \langle v^2\rangle = v_r^2 + v_i^2$$

are written in the angular brackets.

As a rule, the right- and left-sided azimuthal waves are synchronized in amplitude and phase [3] in the linear growth domain; consequently, their superposition yields a wave of the following kind:

$$\{u_r', u_z', \rho', p', s'\} = 2\varkappa\{iv, u, g, \Pi, s\}e^{i\sigma_2}\cos n\varphi,$$
$$u_\varphi' = 2i\varkappa w e^{i\sigma_2}\sin n\varphi, \quad \sigma_2 = \alpha z - \omega t.$$

The Reynolds stress produced by such flapping fluctuations will be periodic in the azimuthal angle with period $T = \pi/n$; for instance,

$$\langle u_r'^2\rangle = 2\varkappa^2 \langle v^2\rangle \exp(-2\alpha_i z)\cos^2 n\varphi, \quad \langle u_\varphi'^2\rangle = 2\varkappa^2 \langle w^2\rangle \exp(-2\alpha_i z)\sin^2 n\varphi,$$

$$\langle u_r' u_z'\rangle = 2\varkappa^2 \langle vu\rangle \exp(-2\alpha_i z)\cos^2 n\varphi, \quad \langle u_r' u_\varphi'\rangle = \varkappa^2 \langle vw\rangle \exp(-2\alpha_i z)\sin 2n\varphi.$$

The evolution of the mean flow is studied on the basis of numerical integration of the system of averaged equations of motion (the Euler system), the continuity and entropy conservation equations. This system is written in the form of conservation laws as

$$\frac{\partial F}{\partial t} + \frac{\partial K}{\partial r} + \frac{1}{r}\frac{\partial L}{\partial \varphi} + \frac{\partial N}{\partial z} + \frac{Q}{r} = 0,$$

where

$$F = |\rho,\ \rho U,\ \rho V,\ \rho W,\ \rho S|;$$

$$K = \begin{vmatrix} \rho U + \langle \rho' u_r'\rangle \\ \rho U^2 + P + \rho\langle u_r'^2\rangle + 2U\langle \rho' u_r'\rangle \\ \rho UV + \rho\langle u_r' u_\varphi'\rangle + U\langle \rho' u_\varphi'\rangle + V\langle \rho' u_r'\rangle \\ \rho UW + \rho\langle u_r' u_z'\rangle + U\langle \rho' u_z'\rangle + W\langle \rho' u_r'\rangle \\ \rho US + \rho\langle u_r' s'\rangle + U\langle \rho' s'\rangle + S\langle \rho' u_r'\rangle \end{vmatrix};$$

$$L = \begin{vmatrix} \rho V + \langle \rho' u_\varphi'\rangle \\ K[3] \\ \rho V^2 + P + \rho\langle u_\varphi'^2\rangle + 2V\langle \rho' u_\varphi'\rangle \\ \rho VW + \rho\langle u_\varphi' u_z'\rangle + V\langle \rho' u_z'\rangle + W\langle \rho' u_\varphi'\rangle \\ \rho VS + \rho\langle u_\varphi' s'\rangle + V\langle \rho' s'\rangle + S\langle \rho' u_\varphi'\rangle \end{vmatrix};$$

$$N = \begin{vmatrix} \rho W + \langle \rho' u_z'\rangle \\ K[4] \\ L[4] \\ \rho W^2 + P + \rho\langle u_z'^2\rangle + 2W\langle \rho' u_z'\rangle \\ \rho WS + \rho\langle u_z' s'\rangle + W\langle \rho' s'\rangle + S\langle \rho' u_z'\rangle \end{vmatrix};$$

$$Q = \begin{vmatrix} K[1] \\ K[2] - L[3] - F_1 \\ 2K[3] \\ K[4] - F_2 \\ K[5] \end{vmatrix};$$

F_1 and F_2 are the equivalent of viscous forces realizing spreading of the jet. The numerical integration is by the MacCormack scheme (explicit second-order difference scheme [9]). The distributions U_0, W_0, ρ_0 as well as $V_0 = 0$, $P_0 = 1$, $S_0 = -\ln(\rho_0)$ were taken as initial values.

TABLE 1

δ	z	α_r	α_i	$\exp(-2\alpha_i z)$	α_r	α_i	$\exp(-2\alpha_i z)$	α_r	α_i	$\exp(-2\alpha_i z)$
		$n=0$			$n=1$			$n=2$		
0,20	0,8768				1,7834	−0,9080	4,9147			
0,25	1,0960				1,8739	−0,8742	6,7957			
0,30	1,3152	1,8397	−0,7088	6,4525	1,9697	−0,8199	8,6413	2,0862	−0,8595	9,5902
0,35	1,5342	1,9200	−0,6292	6,8963	2,0643	−0,7377	9,6200	2,2026	−0,7390	9,6600
0,40	1,7536	1,9844	−0,5193	6,1803	2,1444	−0,6225	8,8735	2,2949	−0,5750	7,5131
0,45	1,9728	2,0131	−0,3848	4,5645	2,1900	−0,4785	6,6070	2,3349	−0,3765	4,4169
0,50	2,1920				2,1852	−0,3278	4,2084			

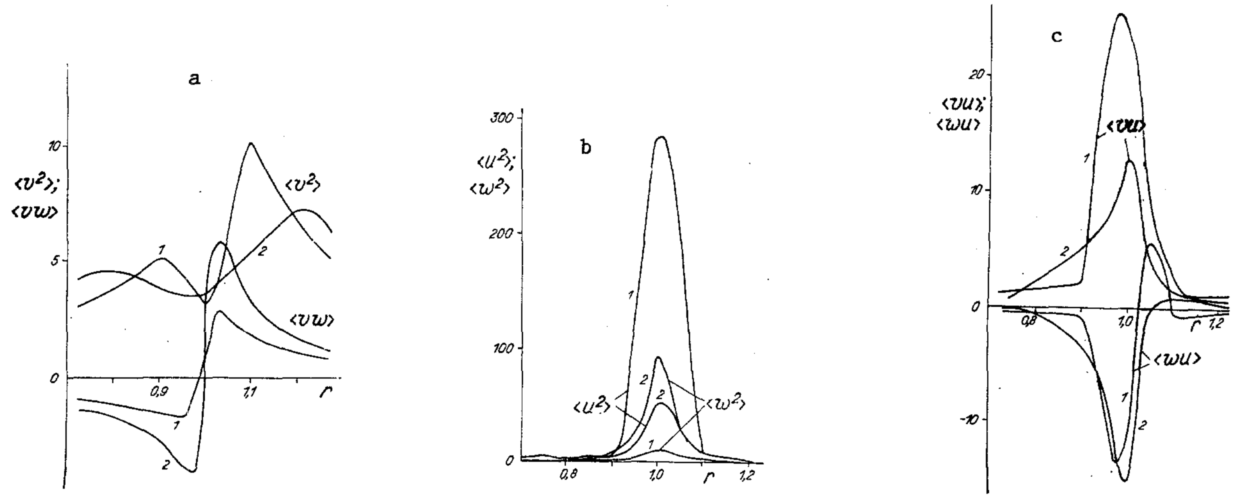

Fig. 2

A variant of flow and wave parameters convenient for numerical integration within the limits of the formulated purpose of clarifying the qualitative features of the expanding flow structure was taken for the computation. This is, first, the regime of moderate supersonic velocities $M_0 = 1.5$, which permitted us to limit ourselves to a linear approximation for the wave analysis; second, moderate frequencies Sh = 0.4 (Sh = $\omega\bar{r}/\pi\bar{W}$) from the domain of the most unstable for a given Mach number; third, the range of mixing layer thicknesses $0.2 \leq \delta \leq 0.5$, within whose limits the waves grow, reach saturation, and start to damp out; and, finally, the value $\kappa = 0.005$ for this normalization of the waves (max $|\Pi| = 1$) yields its intensity <10% of the mean velocity W_0.

Presented in Table 1 are values of the parameters used in the computation.

Amplitude functions of certain Reynolds stress components are shown in Fig. 2 for the limit values $\delta = 0.2$ and 0.5 (lines 1 and 2, respectively) of the flapping-type perturbations $n = \pm 1$ (a is the amplitude of $\langle v^2\rangle$ and $\langle vw\rangle$; b of $\langle w^2\rangle$ and $\langle u^2\rangle$; c of $\langle vu\rangle$ and $\langle wu\rangle$). Because of the diminution of the mean velocity gradients in the mixing layer, smoothing of the appropriate amplitude stress functions occurs downstream, with the exception of the quantities associated with the component u_φ'. Such a nature of the behavior of the additional terms of the system results in the fact that it is difficult to extract the principal terms in it that influence the redistribution processes for different δ.

Let us consider the action of unit waves. Shown in Fig. 3 in the form of the velocity defect $\Delta W = W - W_0$ are the changes which the longitudinal component of the mean profile can

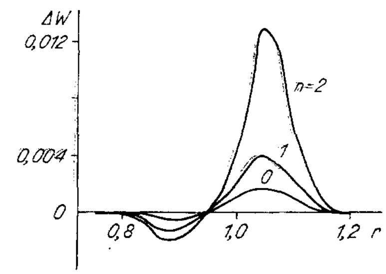

Fig. 3

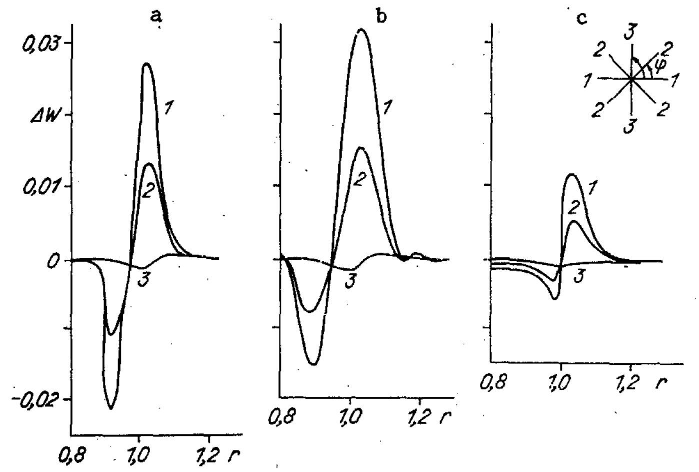

Fig. 4

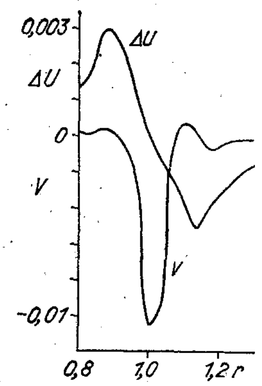

Fig. 5

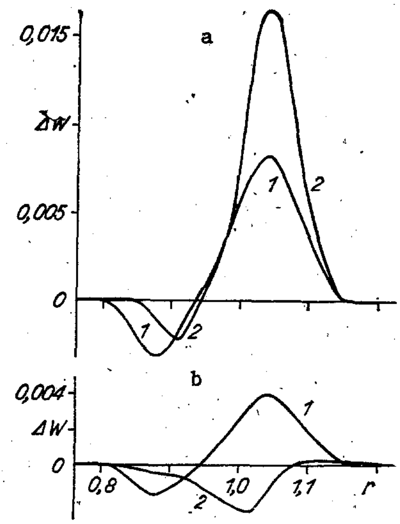

Fig. 6

undergo under the effect of waves in one section (z = const) at the time $t = 0.2$. It is seen that a homogeneous roller or crest occurs for all angles φ, which can be considered as the initial shape of the secondary flow structure being generated. All the velocity changes take place in the mixing layer; the section $\delta = 0.35$ is given in Fig. 3, in which they are maximal. The dynamics of strain growth is in complete conformity with the data of Table 1, it increases from the initial section, reaches the maximum for $\delta = 0.35$, and later starts to decrease downstream according to $\exp(-2\alpha_i z)$. For equality of the amplitude parameter κ the waves of the spiral modes cause a large deformation of the velocity field W, which increases as the azimuthal number n grows.

The unit axisymmetric fluctuations for which $u_\varphi' = 0$ do not induce a tangential component V, it appears for the spiral modes.

The velocities are deformed more complexly under the action of the flapping fluctuations, the azimuthal dependences are tracked clearly here, their repeatability is determined by the quantity n; thus, for $n = \pm 1$, the repetition rate is $T = \pi/2$, while for $n = \pm 2$, $T = \pi/4$. A typical pattern of the change in W in the investigated domain δ is presented in Fig. 4 ($\delta = 0.2, 0.35, 0.5$, a-c) for $n = \pm 1$; $\varphi = 0, \pi/4, \pi/2$ (lines 1-3) at the time $t = 0.5$. The dynamics of the deformation of W downstream reflects the longitudinal evolution of the wave and the diminution of the mean gradients while the tangential changes are related to the initial periodicity of the waves and the Reynolds stresses in φ. Such a wave produces rollers and troughs in the stream, which respectively accelerated and decelerate it in different azimuthal positions. This is reflected here schematically. Azimuthal motion of the gas mass also occurs at certain azimuthal positions; it is absent for the parameters under consideration in positions 1 and 3, masked completely by the radial spreading, and it is maximal at positions 2. This results in moderate twisting of the flow, and the maximal angle of deviation of the velocity vectors U and V reaches 10-13°. The representation of such an additional deformation of the fields U and V is given in Fig. 5, where the velocity V and the defect $\Delta U = U - U_0$ are shown at the position $\delta = 0.35$, $\varphi = \pi/4$ for the same time.

Presented in Fig. 6 are the comparative deformations producible by the modes $n = \pm 1$ and ± 2 (lines 1 and 2) at different azimuthal positions $\varphi = 0$ and $\pi/4$ (a, b) for the same δ and $t = 0.1$. Exactly as for unit fluctuations, the maximal changes are associated with waves of higher azimuthal modes although the action of the latter is more local because of their high periodicity. The characteristics ρ and S undergo similar changes. Therefore, flapping type waves can cause a complex mass redistribution in the mixing layer, which will result in a more complex secondary flow structure in the domain of finite amplitude wave action.

As a rule, perturbations consisting of axisymmetric $n = 0$ and flapping $n = \pm 1$ fluctuations [3], are identified in the spectrum of a naturally excited jet, and their action on the flow, depending on the amplitudes and quantitative composition of the wave, will be comprised of the simpler actions examined above.

The numerical modeling performed shows that large-scale finite-intensity fluctuations will result in finite deformations of the flow characteristics. Such deformations can be detected by careful and purposeful experimental measurement in different azimuthal planes. The appearance of such a fine secondary structure in the flow should undoubtedly affect even the production of jet noise.

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